Practice Set I - Due Feb 14th

Covers §1.1-§1.3, §2.1-§2.3

January 31, 2019

1. Verify the giving function is an explicit solution/a family of solution to the DE.
   (a) $\frac{dy}{dx} = 25 + y^2$, $y = 5\tan(5x)$.
   (b) $y' + 4xy = 8x^3$, $y = 2x^2 - 1 + C_1 e^{-2x^2}$.
   (c) $y'' - 4y' + 4y = 0$, $y = C_1 e^{2x} + C_2 x e^{2x}$.
2. Determing the following statement is True or False, and briefly explain your reason.
   (a) The DE $y' + xy = \sin x$ has a two-parameter family of solution $y = C_1 x \sin x + C_2 \sin x$.
   (b) The solution to the DE $\frac{dy}{dx} = x(1-x)e^{-x}$ is increasing when$x > 1$.
   (c) The function $y = \sin x$ is an explicit solution to the DE $y'' + y = 0$, and the interval $(0, 2\pi)$ is an interval of solution.
3. Draw the phase portrait of the DE $\frac{dy}{dx} = y^2(4-y)$, and if a solution to the DE satisfies the initial condition $y(4) = 2$, then what is the range of this solution?
4. Find all solutions to the following differential equations by separation of variables.
   (a) $\frac{dT}{dt} = kT$, where $k \neq 0$ is an constant and $T > 0$.
   (b) $y' = x\sqrt{x^2 + 1}$.
   (c) $dx + e^{x+2y} dy = 0$.
5. Find all solutions to the following linear differential equations by multiplying an integrating factor.
   (a) $\frac{dy}{dx} + 4x^3 y = x^3$.
   (b) $xy' + y = \sin 2x$.
6. Find all solutions to the following DEs in two ways: separation of variable, and multiplying an integrating factor.
   (a) $\frac{dy}{dx} = 5y$.
   (b) $y' + 3x^2 y = x^2$.
7. Solve the following Initial Value Problem and point out the interval of solution:
   (a) $\frac{dy}{dx} = \frac{1}{y}$, $y(0) = 1$.
   (b) $y' = -xe^y$, $y(10) = 1$.
8. Cooling of a cake: When a cake is removed from an oven, its temperature is measured at 300F. The room temerature is 65F. We know the cooling constant is $k = \frac{1}{3}\ln\frac{13}{23}$. How long will it take for the cake to cool off to a temperature of 150F? (Newton's Law of Cooling)

9. Growth and Decay: The population of a community is known to increase at a rate proportional to the number of people present at time t. If an initial population P_0 has doubled in 5 years, how long will it take to triple?

10. Challenge Questions: verify solution. (Won't count for completeness and correctness purpose!)

    (a) $\frac{dy}{dx} = (y-1)(1-2y)$, $\ln\left(\frac{2y-1}{y-1}\right) = x$.

    (b) $x\frac{dy}{dx} + y = \frac{1}{y^2}$, $x^3y^3 = x^3 + 1$.

    (c) $\frac{dy}{dx} - 2xy = e^x$, $y = e^{x^2}\int_0^x e^{t-t^2}\,dt$.