Common Algebra Fact for Math 2410Q

This sheet only includes the most common algebra facts that will be used in Math 2410Q due to the author's experience. For more comprehensive algebra references, you can easily find it by googling "algebra cheat sheet", e.t.c.

1 Fractions

- identity:

$$1 = \frac{a}{a}, \quad b = \frac{ba}{a}.$$

- common denominator:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}, \quad \text{because} \quad \frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{cb}{bd} = \frac{ad+bc}{bd};$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}, \quad \text{because} \quad \frac{a}{b} - \frac{c}{d} = \frac{ad}{bd} - \frac{cb}{bd} = \frac{ad-bc}{bd}.$$

- fraction of fractions:

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}, \quad \text{because} \quad \frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{bc};$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}, \quad \text{because} \quad \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

2 Exponent Properties

- miltiplication and fraction:

$$a^n a^m = a^{n+m}, \quad \frac{1}{a^n} = a^{-n},$$

$$\frac{a^n}{a^m} = a^n \div \frac{1}{a^m} = a^n \cdot a^{-m} = a^{n-m},$$

- exponent of exponent:

$$(a^n)^m = a^{nm}, \quad a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (a^m)^{\frac{1}{n}}.$$

- specially:

$$a^{\frac{1}{n}} = \sqrt[n]{a}, \quad a^0 = 1.$$

3 Logarithm Properties

- definition: $x > 0$,

$$\text{natural logarithm:} \quad y = \ln x = \log_e x \Leftrightarrow x = e^y.$$

$$\text{In general:} \quad y = \log_b x \Leftrightarrow x = b^y.$$

- properties:

$$\ln(xy) = \ln x + \ln y, \quad \ln\left(\frac{x}{y}\right) = \ln x - \ln y,$$

$$\ln x^y = y \ln x, \quad e^{\ln x} = x, \quad \ln 1 = 0.$$

4 Quadratic Equation

- the quadratic equation ($a \neq 0$)

$$ax^2 + bx + c = 0$$

has two solutions

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

5 Common Factoring Formulas

- $x^2 - y^2 = (x - y)(x + y)$,
- $x^2 + 2xy + y^2 = (x + y)^2$,
- $x^2 + (a + b)x + ab = (x + a)(x + b)$.

6 Complex Numbers

- (unit) imaginary number: $i = \sqrt{-1}$, $\quad i^2 = -1$.
  - e.g.: $\sqrt{-2} = \sqrt{2}i$, $\sqrt{-a} = \sqrt{a}i$ for $a \geq 0$.
- properties: a, b, c, d are real numbers
  - $(a + bi) + (c + di) = (a + b) + (c + d)i$,
  - $(a + bi)(c + di) = ac + bd + (ad + bc)i$,
  - $(a + bi)(a - bi) = a^2 - b^2$.
- conjugate: if $x = a + bi$ is a complex number, the conjugate of x is

$$\overline{x} = \overline{a + bi} = a - bi.$$