Practice Sheet - Week 1

August 27, 2019

1. Determine which of the followings are differential equations, and state your reason.

   (a) $y' = 0$ (b) $y^{(6)} - y^6 = 0$ (c) $f^2 f^{(2)} = x$ (d) $\frac{dy}{dx} = 1$ (e) $y' + x$

2. Determine the order of the following differential equations. Also determine if it is linear or nonlinear.

   (a) $ty''' - y'' - 2y = 0$ (b) $\frac{dy^6}{dt^6} - 2\frac{dy}{dt} + y = t^2$ (c) $y^{(3)} + (y')^3 = x + 1$ (d) $\cos y + y' = t$

3. Find the number of parameters that the family of solution has for the following D.E.

   (a) $y'' + 2y' + 4y = 5$,

   (b) $x^3 y''' + 2x^2 y'' + 20xy' - 78y = 0$,

4. Consider the given 3 - parameter family function $y = C_1 e^x + C_2 e^{-x} + C_3$, which one of the following D.E.s has the given family function as its general solution?

   (a) $y'' - y' = 0$ (b) $y'' - 1 = 0$ (c) $y''' - y' = 0$ (d) $y^3 - y = 0$

5. Determine the following statement is True or False, and briefly explain your reason.

   The DE $y' + xy = \sin x$ has a two-parameter family of solution $y = C_1 x \sin x + C_2 \sin x$.

6. Verify $y = 5\tan(5x)$ is an explicit solution to the D.E. $\frac{dy}{dx} = 25 + y^2$.

7. Check if the function $y(t) = t + 1$ is a solution to the following differential equation:

$$\frac{dy}{dt} = \frac{y^2 - 1}{t^2 + 2t}.$$

8. Verify the given function is a family of solution to the D.E.

   (a) $y' + 4xy = 8x^3$, $y = 2x^2 - 1 + C_1 e^{-2x^2}$.

   (b) $y'' - 4y' + 4y = 0$, $y = C_1 e^{2x} + C_2 x e^{2x}$.

9. Consider the initial value problem

$$\begin{cases} \dfrac{dy}{dx} = y(1-y), \\ y(0) = \dfrac{2}{3}. \end{cases}$$

   We know the function $y = \frac{Ce^x}{1+Ce^x}$ is a family of solution to the DE $y' = y(1-y)$, find an explicit solution to the I. V. P.

10. Rewrite the following D.E.s in differential form, and find the functions $M(x, y)$ and $N(x, y)$ in the definition.

(a) $y\cos x + (y + \sin x)y' = 0$ (b) $\frac{dy}{dx} = \frac{y}{x+y}$

11. Verify that the relation $x^2 + y^2 = 4$ is an implicit solution of the D.E.

$$\frac{dy}{dx} = -\frac{x}{y}.$$

12. A box of peanut butter ice cream is removed from a freezer where the temperature is 15^oF, and then it is placed on a table in a room where the temperature is 68^oF. After 30 min, the temperature of the ice cream is increased to 45^oF. By Newton's law of warming/cooling, the rate at which the temperature of a subject changes is proportional to the difference between the temperature of the subject and the tempertaure of the surrounding. Set up a math model by using differential equation.

13. The population of bacteria in a culture grows at a rate **proportional** to the number of bacteria present at time t. Set up a differential equation to represent the relation between the rate of grow and the population of the bacteria.

14. Challenge Questions (Won't be on Quiz): verify solution.

    (a) $\frac{dy}{dx} = (y-1)(1-2y)$, $\ln\left(\frac{2y-1}{y-1}\right) = x$.

    (b) $x\frac{dy}{dx} + y = \frac{1}{y^2}$, $x^3y^3 = x^3 + 1$.

    (c) $\frac{dy}{dx} - 2xy = e^x$, $y = e^{x^2}\int_0^x e^{t-t^2}\,dt$.