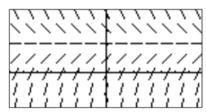
Practice Sheet - Week 2

September 3, 2019

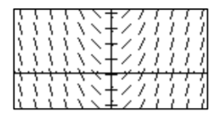
1. (Question 12 from Week 1)A box of peanut butter ice cream is removed from a freezer where the temperature is 15^oF, and then it is placed on a table in a room where the temperature is 68^oF. After 30 min, the temperature of the ice cream is increased to 45^oF. By Newton's law of warming/cooling, the rate at which the temperature of a subject changes is proportional to the difference between the temperature of the subject and the tempertaure of the surrounding. Set up a math model by using differential equation.

2. (Question 13 from Week 2)The population of bacteria in a culture grows at a rate **proportional** to the number of bacteria present at time t. Set up a differential equation to represent the relation between the rate of grow and the population of the bacteria.

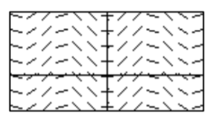
(A)



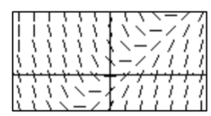
(B)



(C)



(D)



3. Match the following slope fields with their equations

   (a) $\frac{dy}{dt} = \sin t$

   (b) $\frac{dy}{dt} = t - y$

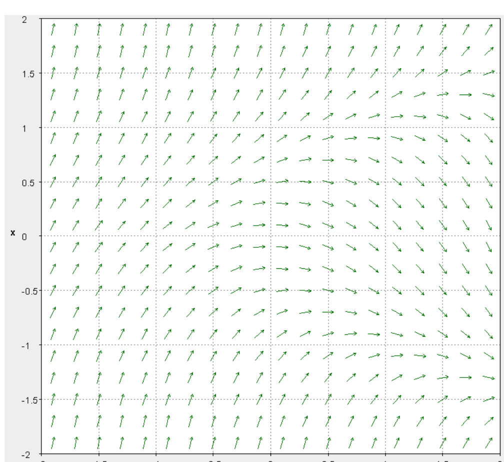
   (c) $\frac{dy}{dt} = 2 - y$

   (d) $\frac{dy}{dt} = t$

4. Suppose the following ODE

$$\frac{dy}{dt} = y^2 - t$$

has the following Slope Field:



(a) Suppose $y(t)$ is a solution to this ODE and also you know that $y(-1) = 1$. Then based on the slope field, what is your prediction for the long term behavior of $y(t)$, that is, what is your prediction of

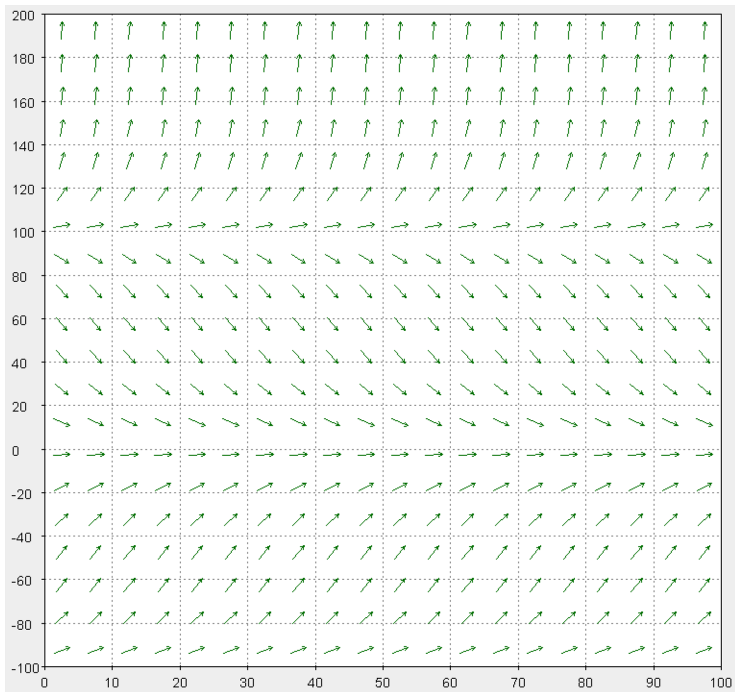
$$\lim_{t\to\infty} y(t) = ?$$

(b) Suppose $y(t)$ is a solution to this ODE and also you know that $y(1) = 0$. Then based on the slope field, what is your prediction for the long term behavior of $y(t)$, that is, what is your prediction of

$$\lim_{t\to\infty} y(t) = ?$$

5. Let $P(t)$ represent the population of the Puffy dog breed. Suppose you come up with the following differential equation that models $P(t)$:

$$\frac{dP}{dt} = P(P-100)(P+100)/100000$$



Its Slope Field is given by:

Suppose that the population of the Puffy dog is 80 at time $t = 0$. What is the long term behavior for the population of the Phan fish? Will it keep increasing/decreasing, stabilize to a certain number, or go extinct?

6. Draw the phase portrait of the DE $\frac{dy}{dx} = y^2(4-y)$, describe the proporties of all possible solutions of the D.E.

7. Find all solutions to the following differential equations by separation of variables.

(a) $\frac{dT}{dt} = kT$, where $k \neq 0$ is an constant and $T > 0$.

(b) $y' = x\sqrt{x^2+1}$.

(c) $dx + e^{x+2y}dy = 0$.