

Practice - Week 3

September 9, 2019

1. Solve the following D.E.s by using the method of 1st-order linear D.E.

(a) $\frac{dy}{dx} + 4x^3y = x^3$.

(b) $xy' + y = \sin 2x$.

2. Solve the following D.E.s by using the method of 1st-order linear D.E. and separation of variables.

(a) $\frac{dy}{dx} = 5y$.

(b) $y' + 3x^2y = x^2$.

3. Solve the following D.E.s.

(a) $\left(\frac{3}{y^5}\right) \frac{dy}{dt} + \frac{t}{2y^4} + \frac{5}{y^5} = 0$.

(b) $y' - y = 2te^{2t}$.

(c) $ty' + 2y = \sin t, t > 0$.

4. Solve the following initial value problem:

$$ty' + (t + 1)y = 2te^{-t}, \quad y(1) = a, \quad t > 0.$$

5. Find the general solutions for the following differential equations. (Determine what formula you want to use first!)

(a) $y' = \frac{x^2}{y}$,

(b) $x \frac{dy}{dx} = 2e^x - y + 6x^2$,

(c) $(\cos x - \cos x \sin^2 y)dx + \frac{\cos^2 y}{\sin x} dy = 0$,

(d) $\frac{dy}{dx} = \frac{x^2 - 3}{3 + 2y}$,

(e) $\frac{dy}{dt} = (y + 1)(y - 2)$.

6. Challenge Problem (NOT ON QUIZ OR EXAM). Verify the solution formula for the 1st-order linear D.E., i.e., the following function

$$y(x) = \frac{\int \mu(x)f(x)dx + C}{\mu(x)}, \quad \mu(x) = e^{\int P(x)dx},$$

is a 1-parameter family of solution to the D.E.

$$y' + P(x)y = f(x).$$