

Week 4 Practice

September 17, 2019

1. Show the following differential equations are exact, then solve the D.E. or I.V.P.:

(a) $(\tan x - \sin x \sin y)dx + \cos x \cos y dy = 0$;

(b) $(5y - 2x)y' - 2y = 0$;

2. Find the general solution of the following D.E.s. Each of them is either exact or can be made exact.

(a) $\left(\frac{3y^2 - t^2}{y^5}\right) \frac{dy}{dt} + \frac{t}{2y^4} = 0$;

(b) $y(x + y + 1)dx + (x + 2y)dy = 0$;

(c) $(y^2 + xy^3)dx + (5y^2 - xy + y^3 \sin y)dy = 0$.

3. Solve the following I.V.P.

$$(x + y)^2 dx + (2xy + x^2 - 1)dy = 0, \quad y(1) = 1.$$

4. Solve the following D.E. by using two different methods,

$$x \frac{dy}{dx} = 2e^x - y + 6x^2.$$

5. Solve the following differential equations by using an appropriate substitution (homogeneous, Bernoulli's or reducible):

(a) $x \frac{dy}{dx} + y = \frac{1}{y^2}$;

(b) $\frac{dy}{dx} - y = e^x y^2$;

(c) $\frac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$;

(d) $\frac{dy}{dx} = \frac{x+3y}{3x+y}$;

(e) $-ydx + (x + \sqrt{xy})dy = 0$;

(f) $\frac{dy}{dx} = \frac{1-x-y}{x+y}$.

6. Challenge Question (NOT on Quiz nor Exam) (Recommended for honor program)

(a) Are separable D.E.s exact?

(b) Can we use the method of making non-exact D.E.s exact to dedrive the formula for homogeneous, Bernoulli's and reducible D.E.s? Why or Why not? (Don't try too hard and spend too much time).

(c) Dedrive the formula for homogeneous, and think about why being homogeneous is an important property (you can google it).

(d) Try to dedrive the formula of making non-exact exact on your own (trying it on your own will make a huge difference than just understanding your notes).