

Week 6 - Practice

September 30, 2019

1. Find the Laplace transform $\mathcal{L}\{f(t)\}$ of the following functions by definition,

$$(a) f(t) = \begin{cases} -1, & 0 \leq t < 1, \\ 1, & t \geq 1. \end{cases}$$

$$(b) f(t) = \begin{cases} 2t + 1, & 0 \leq t < 1, \\ 0, & t \geq 1. \end{cases}$$

$$(c) f(t) = e^{t+7},$$

$$(d) f(t) = \cos t.$$

2. Find the Laplace transform $\mathcal{L}\{f(t)\}$ of the following functions by the following chart

$$\begin{array}{lll} 1) \mathcal{L}\{1\} = \frac{1}{s}, & 2) \mathcal{L}\{e^{at}\} = \frac{1}{s-a}, & 3) \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \text{ where } n! = 1 \cdot 2 \cdot \dots \cdot n, n = 1, 2, 3, \dots \\ 4) \mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2}, & 5) \mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}, & \end{array}$$

$$(a) f(t) = 7t + 3;$$

$$(b) f(t) = t^2 - e^{-9t} + 5;$$

$$(c) f(t) = 4t^2 - 5 \sin 3t;$$

$$(d) f(t) = \cos 5t + \sin 2t.$$

3. Find the following inverse Laplace transform.

$$(a) \mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\};$$

$$(b) \mathcal{L}^{-1}\left\{\frac{s+1}{s^2-4s}\right\};$$

$$(c) \mathcal{L}^{-1}\left\{\frac{2s-4}{(s^2+s)(s^2+1)}\right\};$$

$$(d) \mathcal{L}^{-1}\left\{\frac{10s}{s^2+16}\right\}.$$

4. Use Laplace transform to solve the following I.V.P.

$$(a) y' - y = 2 \cos 5t, \quad y(0) = 0;$$

$$(b) y' + 6y = e^{4t}, \quad y(0) = 2;$$

$$(c) y' - y = 1, \quad y(0) = 0;$$

5. Challenge: the Laplace transform $\mathcal{L}\{f(t)\}$ of the following functions by definition.

$$(a) f(t) = t \cos t;$$

$$(b) f(t) = t \sin t.$$