Week 7 Practice

October 9, 2019

Formula Chart

- $\mathcal{L}\left\{f(t-a)\mathcal{U}(t-a)\right\} = e^{-as}F(s);$ $\qquad \mathcal{L}\left\{\mathcal{U}(t-a)\right\} = e^{-as}\frac{1}{s}.$
- $\mathcal{L}\left\{e^{at}f(t)\right\} = F(s-a).$
- $\mathcal{L}\left\{t^n f(t)\right\} = (-1)^n \frac{d^n}{ds^n}F(s).$
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  1) $\mathcal{L}\{1\} = \frac{1}{s},$ 2) $\mathcal{L}\{e^{at}\} = \frac{1}{s-a},$ 3) $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}},$ where $n! = 1 \cdot 2 \cdot \dots \cdot n$, $n = 1, 2, 3, \cdots$
  4) $\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2},$ 5) $\mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2},$

1. Rewrite the following functions in terms of the unit step functions, and then use the operational property to find its Laplace transform.

   (a) $f(t) = \begin{cases} t, & 0 \le t < 2, \\ 0, & t \ge 2. \end{cases}$

   (b) $f(t) = \begin{cases} 3, & 0 \le t < 1, \\ 1, & 1 \le t < 6, \\ 6, & t \ge 6. \end{cases}$

2. Find the inverse Laplace transform of the following functions.

   (a) $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\},$

   (b) $\mathcal{L}^{-1}\left\{\frac{e^{-\pi s}}{s^2+1}\right\},$

   (c) $\mathcal{L}^{-1}\left\{\frac{se^{-2s}}{s^2+4}\right\}.$

3. Use the operational property

   $$\mathcal{L}\left\{e^{at}f(t)\right\} = F(s-a)$$

   to find the following Laplace transform or inverse Laplace transform.

   (a) $\mathcal{L}\left\{e^t \sin(3t)\right\},$

   (b) $\mathcal{L}\left\{e^{2t}(t-1)^2\right\},$

   (c) $\mathcal{L}^{-1}\left\{\frac{1}{(s-4)^4}\right\},$

   (d) $\mathcal{L}^{-1}\left\{\frac{5s}{(s-2)^2}\right\},$

4. Find the Laplace transform

   $$\mathcal{L}\left\{t^3 e^{10t}\right\}$$

   by using the operational property

   (a) $\mathcal{L}\left\{e^{at}f(t)\right\} = F(s-a).$

   (b) $\mathcal{L}\left\{t^n f(t)\right\} = (-1)^n \frac{d^n}{ds^n}F(s).$

5. Use Laplace Transform to solve the following I.V.P.

   (a) $y' - y = te^t, \quad y(0) = 0;$

   (b) $y' + y = f(t), \quad y(0) = 0$, where

$$f(t) = \begin{cases} 0, & 0 \le t < 1, \\ 5, & t \ge 1. \end{cases}$$

6. Use Laplace Transform to solve the following the I.V.P.:

$$y'' + 9y = \cos(3t), \quad y(0) = 2, \quad y'(0) = 5.$$

   Hint: use one of the operational property to get $\mathcal{L}\{t\sin(kt)\}$.